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PROJECT REPORT

Introduction to Optimal Control

Title:

Optimal Control of Multiple Inverters Parallel Operation

Submitted By:

Ahmed Tallal

Submitted To:

Prof. Deming Yuan

南京理工大学

NANJING UNIVERSITY OF SCIENCE & TECHNOLOGY

SCHOOL OF AUTOMATION

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OPTIMAL CONTROL OF MULTIPLE INVERTERS PARALLEL OPERATION

Abstract

To ensure the continuous electric supply, uninterruptable power supply systems (UPS) with multiple inverters are widely used. This report presents a optimal control methodology for feed back loops of multiple inverter system. An optimal policy that is independent of the number of inverters is proposed in this study. The proposed controller is designed to reduce the performance measure that is function of inverter currents, reference signals and output voltage error. The robustness of controller is also analyzed by analyzing the current sharing and voltage bode plots. It is seen that by increasing the number of inverters there is little effect on the overall output of the system. The experimental study is also carried out by using three 1.1kVA, 110Vac inverters to further verify the performance of the system under proposed optimal controller.

1. Introduction

The electrical energy is one of the basic requirements for the human beings and had made the life more secure, fast and easy. With the growing population, automation and industry the demand of electricity is also being increased. The supply of electricity may get interrupted by either its unavailability or due to the fault or failure of power supply equipment due to weather or other possible events.

In such cases to ensure the continuous supply of the electricity the battery banks are utilized. For the DC powered load, the battery banks with some voltage regulators are enough but for the AC load inverters are required to for the conversion of DC power to AC power. However, inverters also have their own limitations such as its capacity to handle instantons load and total power. Therefore, the inverters must be capable to increase their power capacity as the load increases. This is only possible either by replacing that inverter or adding another inverter in parallel to share the load, the later one sounds more economic and is widely adopted.

Along with the benefits of parallel operation, there are also some stability issues related to the parallel operation. Various control techniques are being implemented to ensure smooth and stable parallel operation of inverters under varying electrical loads. The most common method is current sharing control is the V and F method, where V corresponds to Voltage and F corresponds to the frequency [1], [2]. A notable advantage of this method is simplicity, because no extra interconnections among inverters are required. However, the voltage regulation and transient responses are sacrificed. In addition, when the load is nonlinear, the harmonic components of the load current cannot be shared properly. A modified frequency and voltage droop method was proposed in [3] to improve performance under nonlinear load, but the voltage regulation is still poor due to the inherent “droop” nature.

During the initial era of the inverter technology because of the limitation of the switching frequency of the switching components, the response of the system is quite show. The shared information is the average root-mean-square (rms) value of the output current or power to compensate the rms value of the output voltage [1], [4], [5]. In such a system, only the rms value of the output current of each inverter is assured to be equal. Under nonlinear loading conditions or load disturbance, the system has poor abilities to balance the harmonics of the output current. The dynamic response of the system is also poor. With the development of semi-conductor technologies, high-speed switching components appeared. To improve the instantaneous current-sharing characteristics, instantaneous current-sharing schemes have been proposed [6]-11]. These methodologies are all based on the instantaneous control of a single inverter and employ some mechanism to share or transfer current-sharing information among the inverters. Since the output currents of the inverters are regulated at every switching cycle, instantaneous current-sharing scheme has very good performance both on current sharing and voltage regulation. Even if the output currents contain harmonics, the inverters can share the output currents equally. However, interconnections between the inverters are necessary. This limits the flexibility of the multi-inverter system and degrades the system’s redundancy.

Parallel operated multiple inverter system is a high order multi-variable system and involves instantaneous current sharing control. For such a complex system, little has been reported on systematic way to optimize the design to achieve the objectives of good voltage regulation and small current unbalance. In this paper, we model the multi-inverter system as a set of multi-variable state-space equations and employ the optimal control methodology [12] to design the feedback loop. The feedback loop thus determined can minimize the current unbalance and reduce the output voltage error. We find that the proposed controller has the following advantages. First, It is robust to the change in the number of inverters. By applying the frequency-domain analysis to a multi-inverter system using the proposed controller, it is found that the performance of the system is practically independent of the number of paralleled inverters. Thus, we can design an optimal controller based on a fixed number of inverters and use it in a system with a variable number of inverters without re-designing. Second, It requires only one common signal line to share the information on the total current. The common circuit of the system is reduced to a current sensor (for every inverter) and a signal wire.

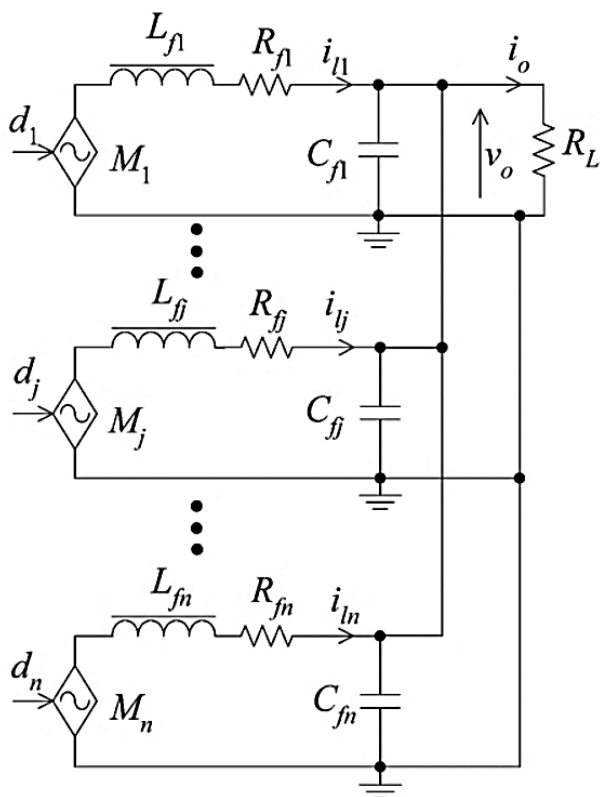

Figure 1: Model of n parallel inverters

2. Modelling of Multiple Inverter System

A model of n parallel multi inverter systems is shown in Figure 1. A sinusoidal pulse-width-modulation (SPWM) inverter can be modeled as a controlled ac voltage source followed by an output LC filter [13]. In this model, $\boldsymbol{M_j}$ is the gain of the SPWM. $\boldsymbol{d_j}$ is the duty ratio of the electronic switches of the inverter. $\boldsymbol{C_{fj}}$, $\boldsymbol{L_{fj}}$, and $\boldsymbol{R_{fj}}$ are the capacitance, inductance, and resistance of the output filter of the **jth** inverter. Assuming a resistive load $\boldsymbol{M_j}$, the state-space form of a multi-inverter system with **n** inverters is written below;

$$\frac{d}{dt}\begin{bmatrix} i_{l1} \\ i_{l2} \\ \vdots \\ i_{ln} \\ v_o \end{bmatrix} = \begin{bmatrix} -\frac{R_{f1}}{L_{f1}} & 0 & \cdots & 0 & -\frac{1}{L_{f1}} \\ 0 & -\frac{R_{f1}}{L_{f1}} & \dots & 0 & -\frac{1}{L_{f2}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\frac{R_{f1}}{L_{f1}} & -\frac{1}{L_{fn}} \\ \frac{1}{C_L} & \frac{1}{C_L} & \cdots & \frac{1}{C_L} & -\frac{1}{C_L R_L} \end{bmatrix} x \begin{bmatrix} i_{l1} \\ i_{l2} \\ \vdots \\ i_{ln} \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{M}{L_{f1}} & 0 & \cdots & 0 \\ 0 & \frac{M}{L_{f1}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{M}{L_{fn}} \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} \quad (1)$$

An integrator is inserted into the system. Let be a state to represent the integration of the output error to improve the steady-state performance of the output voltage;

$$\dot{e}_v = v_r - v_o \quad 2$$

Where v_r is the reference voltage of the inverter system. It can given as;

$$v_r = v_m sin w_r t$$

Therefore, the state space equation for n-paralleled multi0iverter system is;

$$\dot{x} = A_x + B_u + B_w$$

$$y = v_o = C_x \quad 3$$

And;

$$x = \begin{bmatrix} e_v \\ i_{l1} \\ i_{l2} \\ \vdots \\ i_{ln} \\ v_o \end{bmatrix}, \quad u = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & -\frac{R_{f1}}{L_{f1}} & 0 & \cdots & 0 & -\frac{1}{L_{f1}} \\ 0 & 0 & -\frac{R_{f2}}{L_{f12}} & \dots & 0 & -\frac{1}{L_{f2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\frac{R_{fn}}{L_{fn}} & -\frac{1}{L_{fn}} \\ 0 & \frac{1}{C_L} & \frac{1}{C_L} & \cdots & \frac{1}{C_L} & -\frac{1}{C_L R_L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \frac{M}{L_{f1}} & 0 & \cdots & 0 \\ 0 & \frac{M}{L_{f1}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{M}{L_{fn}} \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$B_w = \begin{bmatrix} v_r \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The complete model of multi inverter systems can be described by Equations (1), (3) and Figure 1.

Design of Optimal Controller

Optimal controller is designed to minimize the performance measure. A generic performance measure [12] can be stated as;

$$J_p = \frac{1}{2}\int_0^{\infty} [u^T W_u + X^T Q_1 x + (v_r - y)^T Q_2 (v_r - y)]\, dt \qquad 4$$

Here, $W \epsilon \Re^{nxn}$, $Q_1 \epsilon \Re^{(n+2)x(n+2)}$ and $Q_2 \epsilon\epsilon \Re^{1x1}$ are the positive definite matrixes.

The minimization of the performance measure should be the lowest input energy to the modeled system. The minimization of the output error should be an objective of the system. By adjusting the matrix Q_1, we can determine the different contributions of different states.

In a multi-inverter system, if we select the same weight for the inductor currents of all the inverters, finally all the inductor currents will tend to be the same, so that equal current sharing is achieved. Therefore, we can apply optimal control to tackle both current-sharing and voltage-regulation problem in a multi-inverter system. But the above-mentioned method is applicable to the design of a regulator with dc output only. Since the output voltage of an inverter system should follow a sinusoidal wave, its control becomes a tracking problem. In order to apply optimal control to an inverter system, the first step, therefore, is to reduce this tracking problem to a regulator problem, as elaborated underneath. Starting with the augmented system with the n-paralleled multi-inverter system describes by Equation (3), the voltage reference can be described as;

$$\dot{z} = A_Z z \quad (5)$$

$$\dot{v_r} = C_z^T z \quad (6)$$

$$\dot{z_C} = A_C z_C + B_C u \quad (7)$$

$$J = \frac{1}{2}\int_0^{\infty} [u^T W_u + Z_c^T Q_C z_C \ ]\, dt \quad (8)$$

The control law of the proposed optimal controller can be written as;

$$u = -K_a z_C = -K_a \begin{bmatrix} x \\ z \end{bmatrix} \quad (9)$$

$$= -K_a [e_v \ \ i_{l1} \ \ i_{l2} \dots \ i_{ln} \ \ v_o \ \ v_r \ \ \dot{v}_r]^T \quad (11)$$

Where, $\dot{v}_r$ is the term for augmented system and its coefficient is very small and it can be omitted. The duty ratio of the switches of each inverter in multi-inverter system;

$$d_j = K_{ev} e_v - K_v v_o - \sum_{m=1}^{n} K_{im} i_{lm} - K_r v_r \quad (12)$$

The feedback gains of the inductor currents are equal (K_{ilo}), and the control law can be written as;

$$d_j = K_{ev} e_v - K_r v_r - K_v v_o - K_{ij} i_j - K_{ilo} \sum_{\substack{m=1 \\ m \neq j}}^{n} i_{lm}$$

$$= K_{ev}e_v - K_r v_r - K_v v_o - (K_{ij} - K_{ilo)i_j} - K_{ilo} \sum_{\substack{m=1 \\ m \neq j}}^{n} i_{lm}$$

$$\approx -K_{ev}e_v - K_r v_r - K_v v_o - (K_{ij} - K_{ilo)i_j} - K_{ilo} \left(\sum_{\substack{m=1 \\ m \neq j}}^{n} i_{lm} + C_L \frac{dv_o}{dt} \right)$$

$$= K_{ev}e_v - K_r v_r - K_v v_o - K_i i_j - K_{io} i_o \quad 13$$

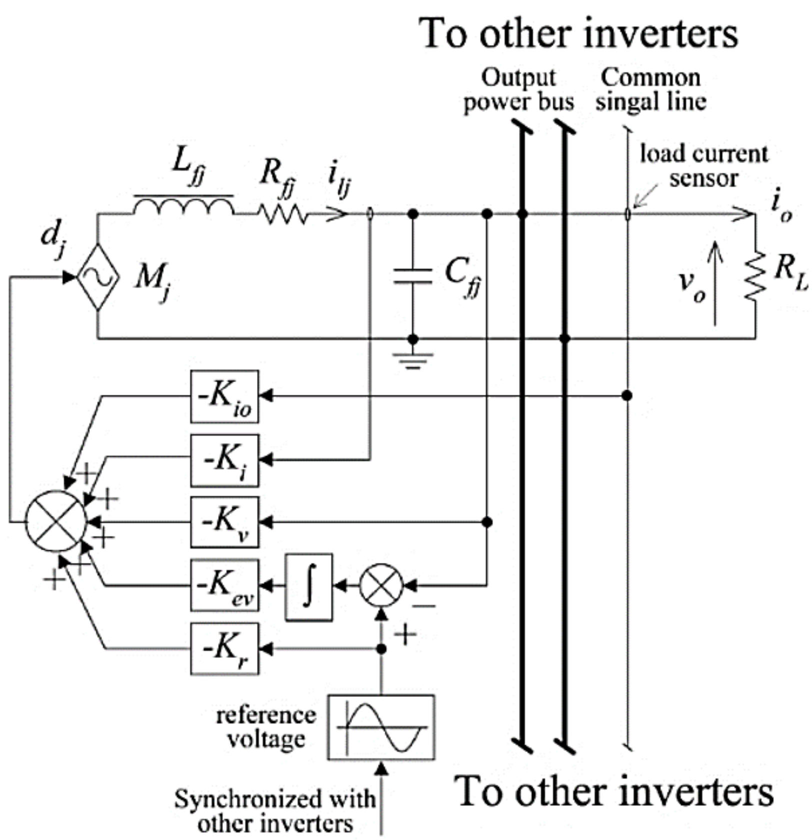

Figure 2: Optimal controller for the jth inverter in multi inverter system design

3. Analysis of Proposed optimal controller

Frequency domain analysis of the multi inverter system is carried out to analyze the robustness of the proposed optimal controller. It is observed that the by increasing or decreasing the number of inverters has no or minimal effect on the multi inverter system's performance.

Multi inverter system in Figure 1 can be described by the following equations;

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_{ij} i_{j1} - K_{io} i_{jo}\right] = i_{l1}\left(L_{f1s} - R_{f1}\right) \quad 14$$

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_{ij} i_{j2} - K_{io} i_o\right] = i_{l2}\left(L_{f2s} - R_{f2}\right)$$

$$\vdots$$

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_i i_{ln} - K_{io} i_o\right] = i_{ln}\left(L_{fns} - R_{fn}\right) \quad 15$$

$$i_{l1} + i_{l2} + \cdots + i_{ln} - v_o C_{Ls} = i_o \quad 16$$

The major concerns regarding the multi-inverter system i-e voltage regulation and current sharing are analyzed in the sections below.

3.1 Investigation of Current Sharing

By assuming the first inverter's parameters to be optimal, the parameters of other inverters are set different. To represent the total effect of deviation a tern Delta_j is introduced. Hence, the system can be described as;

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_i i_{ln} - K_{io} i_o\right] - v_o = i_{ln}\left(L_{fns} - R_{fn}\right)$$

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_i i_{l2} - K_{io} i_o\right] - v_o = i_{l2}\left(L_{fs} - R_f\right) + \text{Delta}_2$$

$$\vdots$$

$$M\left[-\frac{K_{ev}}{s}(v_r - v_o) - K_r v_r - K_v v_o - K_i i_{ln} - K_{io} i_o\right] - v_o = i_{ln}\left(L_{fs} - R_f\right) + \text{Delta}_n$$

$$i_{l1} + i_{l2} + \cdots + i_{ln} - v_o C_{Ls} = i_o \quad 17$$

Solving first equation and equation (17) to get;

$$i_{l1} - i_{lj} = \frac{Delta_j}{L_{fs} + R_f + K_i}, \quad (1 < j \leq n) \tag{18}$$

It is clear that the number of inverters is not contributing in the current unbalance between the first and jth inverter and depends only on deviation from nominal values.

3.2 Analysis of voltage regulation

The voltage regulation for parallel operation of multi-inverters systems is analyzed by the equivalent output impedance. The analysis is based on the assumption that parameters of all the inverters are the same.

By adding summation of first n equations int eh last equation (in 16), the equivalent output impedance of the system is derived as;

$$-\frac{v_o}{i_o}\bigg|_{v_r=0} = \frac{1}{n}\left(\frac{s\left(L_{fs} + K_i + nK_{io} + R_f\right)}{L_f C_s^3 + C_f\left(R_f + K_i\right)S^2 + (K_v + 1)s - K_{ev}}\right) \tag{19}$$

Assuming the Equation (19) denominator to be;

$$D(s) = L_f C_s^3 + C_f\left(R_f + K_i\right)S^2 + (K_v + 1)s - K_{ev}$$

We have;

$$-\frac{v_o}{i_o}\bigg|_{v_r=0} = \frac{K_{io}s}{D(s)} + \frac{s\left(L_{fs} + K_i + R_f\right)}{nD(s)} \tag{20}$$

It can be seen that equivalent output impendence is not strongly inversely proportional to the number of parallel inverters. Here we use Bode Plot to show the relationship between the equivalent output impendence and the number of paralleled inverters. Originally the multi-inverter system is a two-inverter system, where individual inverter has the nominal parameters as listed in Table I, and the control law of (14). We now add new inverters into the system one after another (each assumed to have the same nominal parameters as shown in Table I) and plot the output impedance in a Bode form

The Bode Plot is shown below;

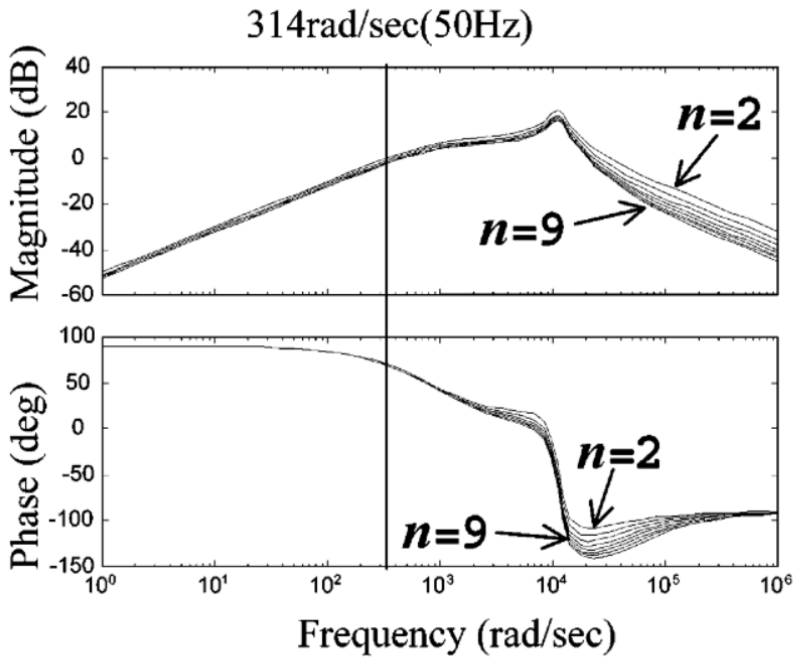

Figure 3: Bode plot of magnitude of the equivalent output impendence of multi inverter system (n=2,---9)

The above figure shows that even with a significant change in the number of parallel inverters, the steady-state error of the output voltage would not vary significantly. In the high-frequency band, the equivalent output impendence varies more significantly. The larger is the number of inverters, the lower is the equivalent output impendence. Lower equivalent output impendence in high-frequency band implies a faster transient response.

4. Experimental Analysis

To further verify the theoretical results, experimental analysis is done by operating three inverters rated 1.1kVA. Each inverter has the parameters as listed in Table I, and the controller has the structure as shown in Fig. 2, and with the parameters of (14). Fig. 4 is the experimental results of the two-inverter system. Fig. 4 is the waveforms when load changes from open circuit (no load) to rated load of 5 . It shows that the transient response is fast. Even under dynamic load, the load current is shared properly. Figs. 5 and 6 are the experimental results of a three-inverter system. Fig. 5 shows the waveforms when the load changes from open circuit (no load) to 5. It shows that, even using the same controller as the two-inverter system, the load current is equally well shared among the three inverters. The response is like that of the two-inverter system shown in Fig. 4. It proves that the controller performs equally well in systems with different number of paralleled inverters. Fig. 6 shows the waveforms when a third inverter is added into the system. Hence, it clear that under all conditions the load current is divided properly.

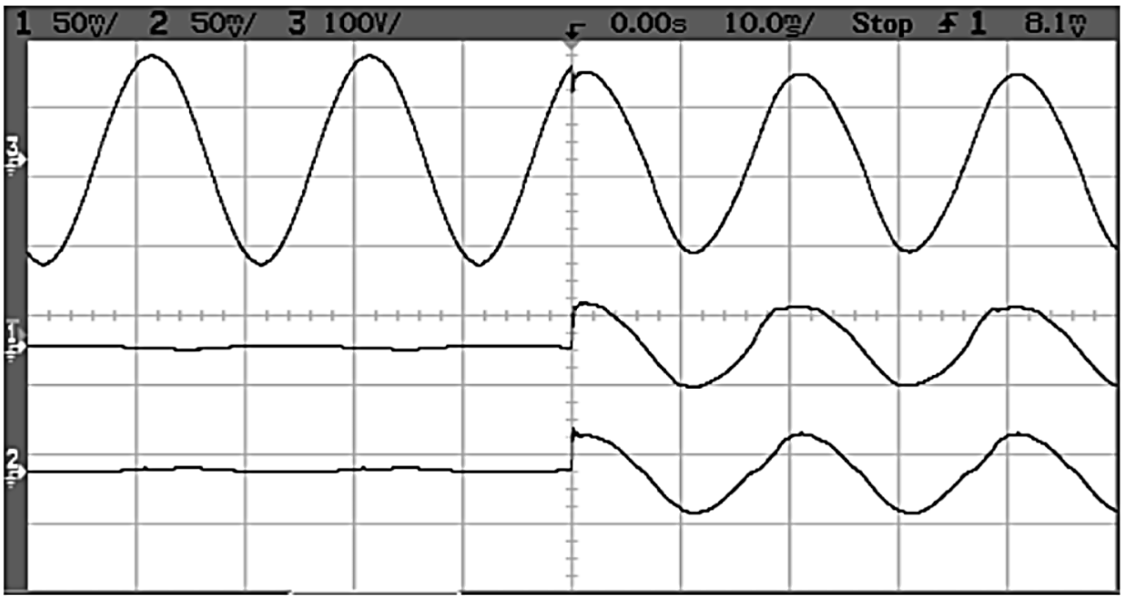

Figure 4: Experimental analysis of two-inverter system when load changes from open circuit (no load) to 5 Ω

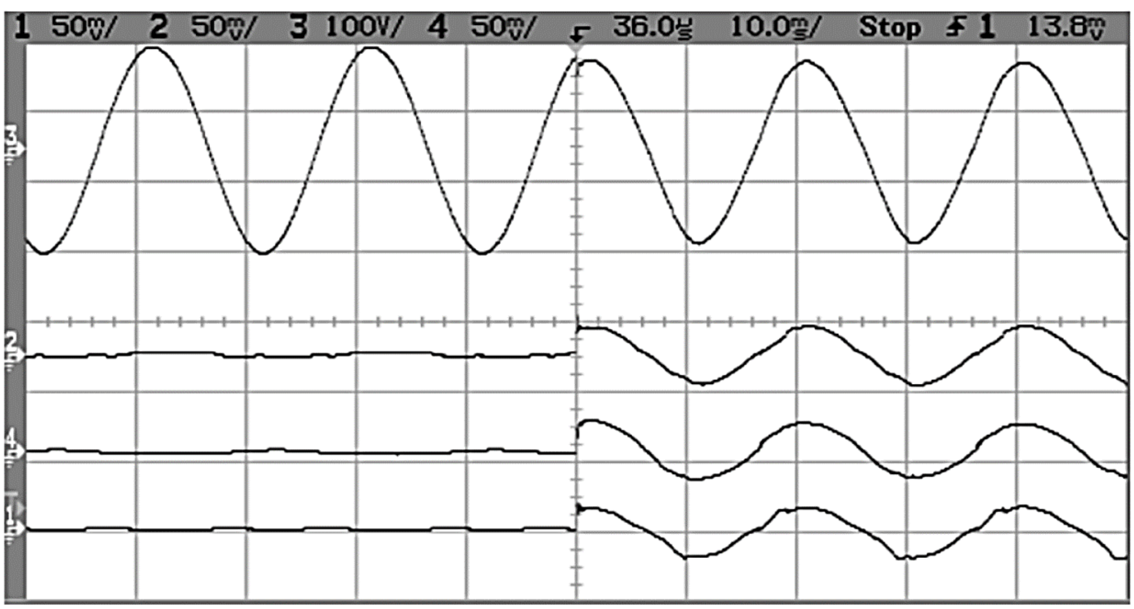

Figure 5: Experimental analysis of a three-inverter system when load changes from open circuit (no load) to 5 Ω

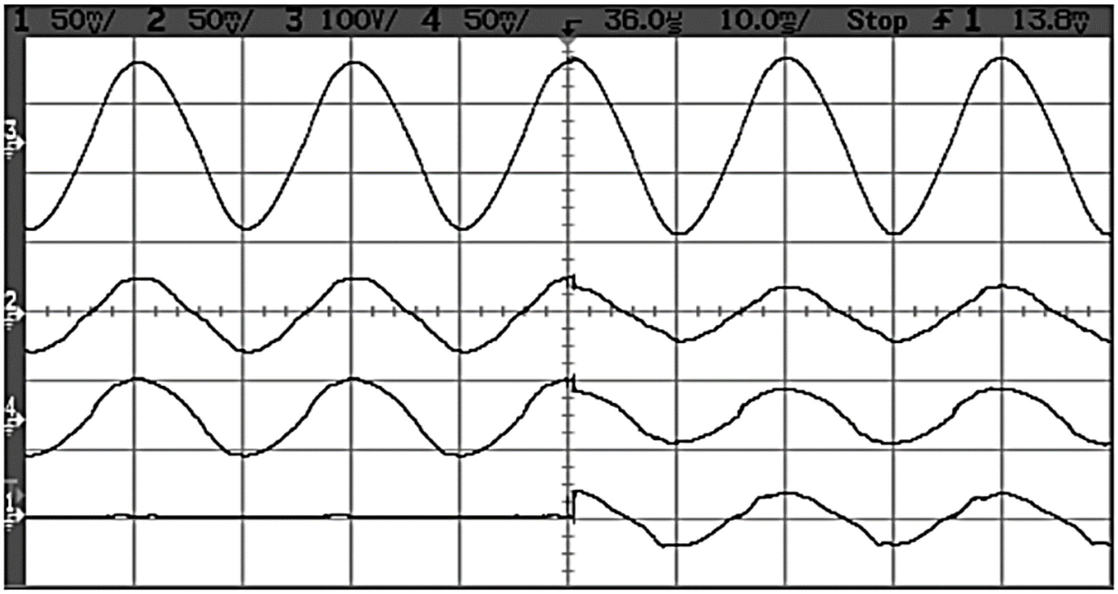

Figure 6: Experimental analysis of a three-inverter system when an inverter is added load of 5Ω

5. Conclusion

In this study optimal control technique is implemented to design optimal controller for a multiple paralleled inverter system. Feedback gain matrix was determined via optimal control methodology. The results indicate that that system voltage and current sharing was stable under the proposed optimal controller. The results were further supported by experimental analysis using 110Vac 3 inverters rated 1.1kVA. The hardware implementation of the circuit is also simple as there is only current sensor for each inverter and single signal wire.

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